個體經濟學一

Microeconomics (I)

Ch4. Comparative Statics and Demand

Equilibrium Analysis {
- Static Analysis：what is an equilibrium conditions of equilibrium.
- Comparative Statics Analysis：compare several equilibria.

Consumer's problem.

$\text{Max } \mathrm{M}(x, y) \quad \text{s.t.} \quad P_x\, x + P_y\, y = m$

FOC $\quad \text{MRS } xy = \frac{P_x}{P_y}$

$P_x\, x + P_y\, y = m$ } (x^*, y^*)

$x^* = x\,(P_x\,,\,P_y\,,\,m)$

$y^* = y\,(P_x\,,\,P_y\,,\,m)$

$P_x\,,\,P_y\,,\,m$ change

$\Rightarrow (x^*, y^*)$ change

4.1 Income change

$m \rightarrow m'$, assume $m' > m$

$P_x\,,\,P_y$ are given.

possible outcome of (x', y')：

$x' > x^*\,,\; y' > y^* \qquad e'$

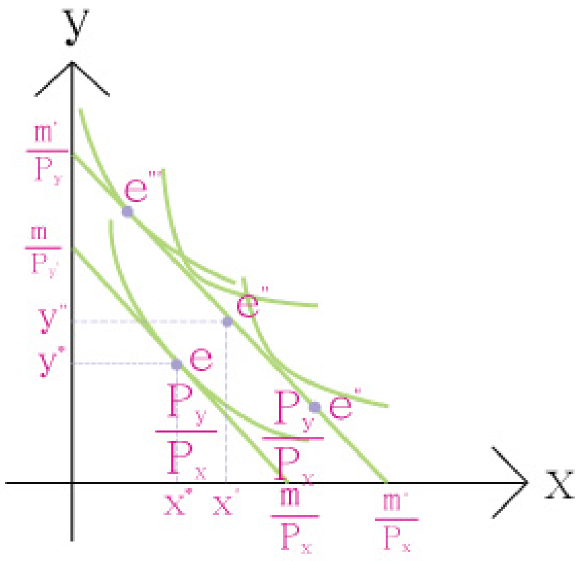
$x' > x^*\,,\; y' < y^* \qquad e''$

$x' < x^*\,,\; y' > y^* \qquad e'''$

$x' < x^*\,,\; y' < y^* \quad \rightarrow$ impossible !

$m\uparrow\,,\; m \rightarrow m' \quad \Rightarrow \quad (x^*, y^*) \rightarrow (x', y')$

(equilibrium $x\uparrow$) 若 $x' > x^*$, we call ***X*** is a normal good.

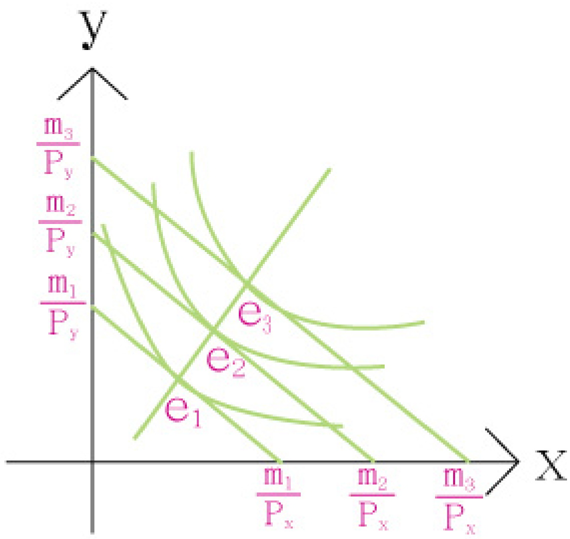


Ｆigure35 :Consumer equilibrium

(equilibrium $y\uparrow$) 若 $y' > y^*$, we call ***Y*** is a normal good.
(equilibrium $x\uparrow$) 若 $x' < x^*$, we call ***X*** is a inferior good.
(equilibrium $y\uparrow$) 若 $y' < y^*$, we call ***Y*** is a inferior good.

劣等財的 IC 仍是負斜率，仍是一個好商品。

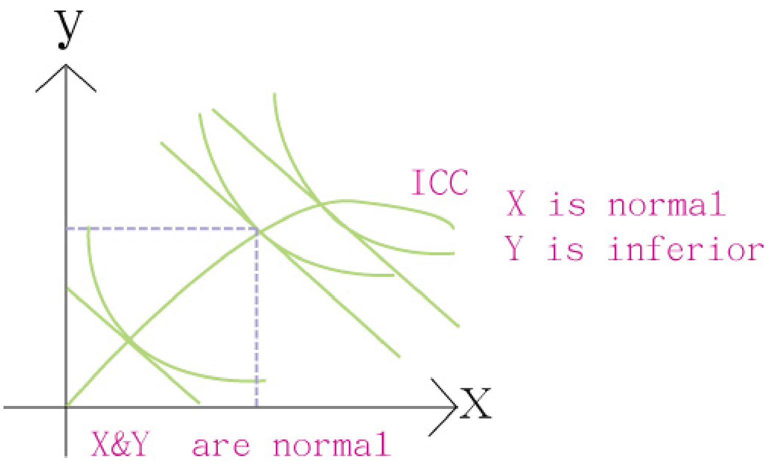
Consider： all possible values of m



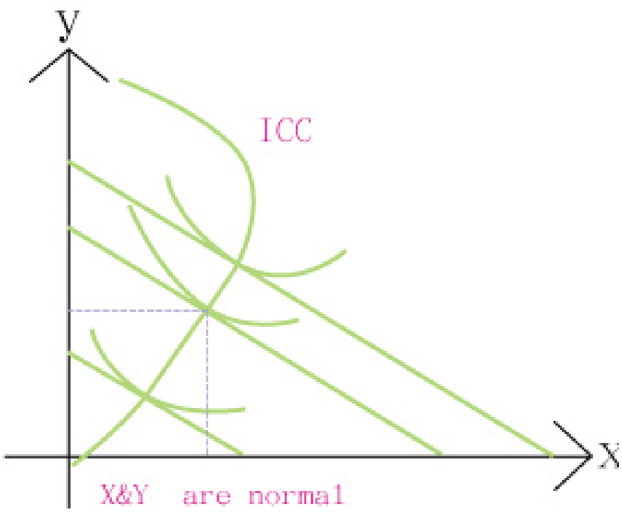
Ｆigure36: Income consumption curve (ICC)

locus of all equilibria with respect to different values of income. （所有相切點的連線）

both *X* & *Y* are normal , ICC is upward sloping.

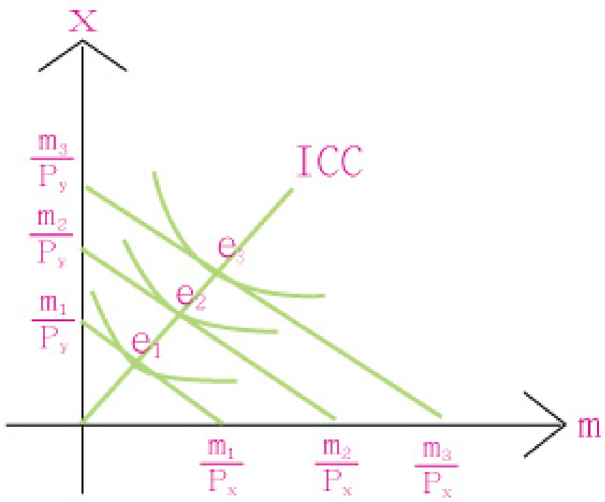


Ｆigure37 :Icc when *x* is normal , *y* is inferior.



Ｆigure38 :Icc when *x* is inferior , *y* is normal.

ICC：Income Consumption Curve



Ｆigure39 : ICC：Income Consumption Curve

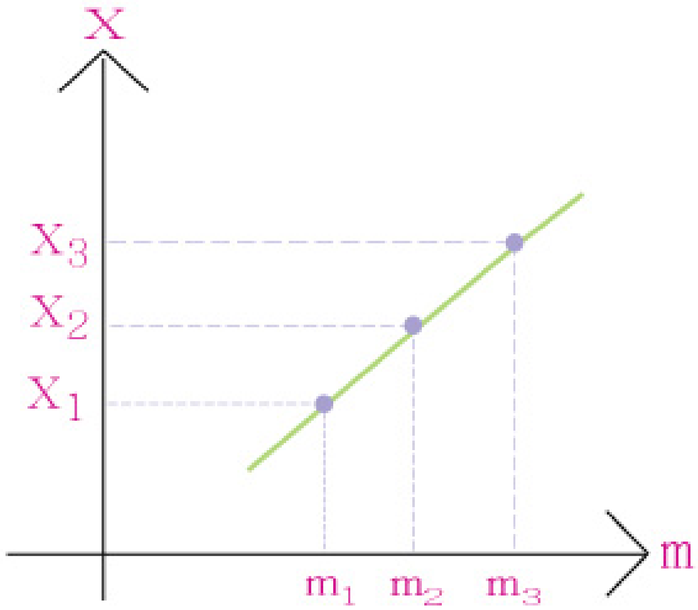


Figure40: Engel curve 均衡&所得的關係

$u(x,y) = x^2 + 0.5y^2$ ⟶ violates diminishing MRS xy (convexity)

EX： $u(x,y) = 2\sqrt{x} + \sqrt{y}$ ， $Max_{x,y}\ 2\sqrt{x} + \sqrt{y}$ $s.t.\ P_x x + P_y y = m$

FOC ⇒ $MRS_{xy} = \frac{P_x}{P_y}$ ------ ①

$P_x x + P_y y = m$ ------ ②

$$MRS_{xy} = \frac{MU_x}{MU_y} = \frac{x^{-0.5}}{0.5y^{-0.5}} = \frac{y^{0.5}}{0.5x^{0.5}}$$

① ⇒ $\frac{2y^{0.5}}{x^{0.5}} = \frac{P_x}{P_y}$ (tangency condition)

$$\frac{4y}{x} = \frac{{P_x}^2}{{P_y}^2}$$

$y = \frac{{P_x}^2}{4 * {P_y}^2} x$ ------ ③

③ ⟶ ② $P_x x + \frac{{P_x}^2}{4P_y} x = m$

$$\frac{4P_xP_y + {P_x}^2}{4P_y} x = m$$

$x^* = x(P_x, P_y, m)$ ， $x = \frac{4P_y}{4P_xP_y + {P_x}^2} m$ ------ ④

③ ⇒ $y^* = y(P_x, P_y, m)$ ， $y = \frac{P_x}{P_xP_y + {P_y}^2} m$ ------ ⑤

Using ④ & ⑤ to eliminate m ⇒ ICC

$$\frac{⑤}{④} \quad \frac{y}{x} = \frac{\frac{P_x}{P_xP_y+P_y{}^2}m}{\frac{4P_y}{4P_xP_y+P_x{}^2}m} = \frac{P_x{}^2}{4\,P_y{}^2}$$

$$\Rightarrow\ y = \frac{P_x{}^2}{4\,P_y{}^2}x.\ \text{(ICC)}$$

Engel curves

$\mathrm{x} = \frac{4\mathrm{P_y}}{4\mathrm{P_xP_y} + \mathrm{P_x}{}^2}\mathrm{m}$, $\mathrm{y} = \frac{\mathrm{P_x}}{\mathrm{P_xP_y} + \mathrm{P_y}{}^2}\mathrm{m}$ are two straight lines from the origin

with slopes s and t.

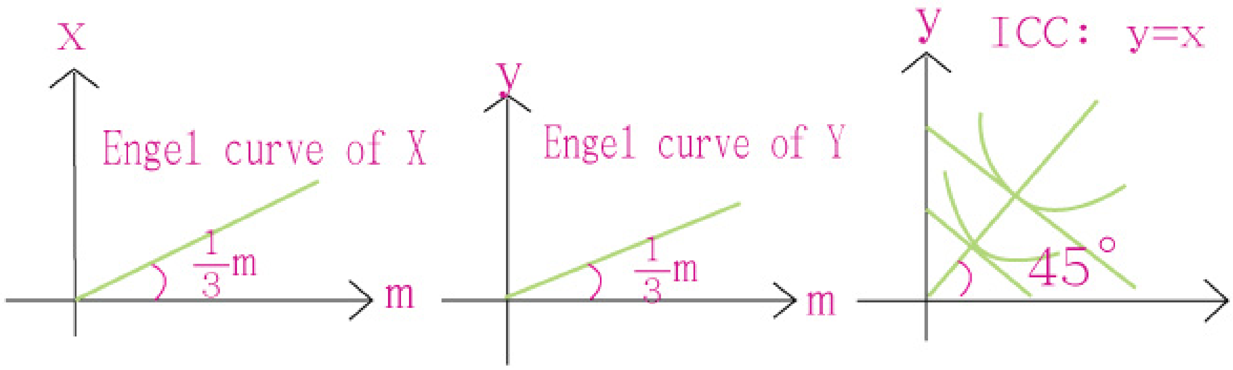


Figure41 :. Ex. $P_x = 2\,, P_y = 1$

Engel curves : $\frac{4}{12}m\,, \mathrm{y} = \frac{2}{6}m$

Another comparative statics analysis：

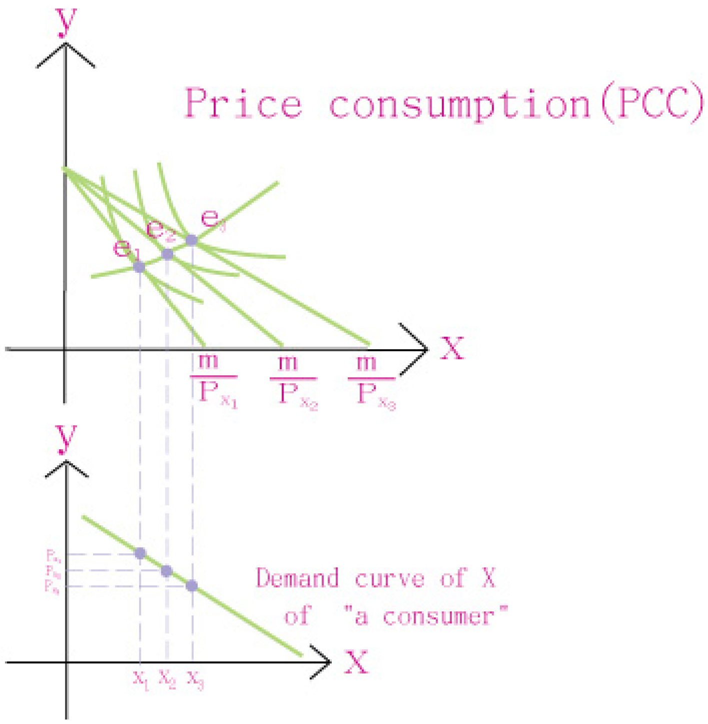


Figure42 : P_x changes (or P_y changes)

$$P_x \downarrow \;,\; P_{x1} \rightarrow P_{x2} \;,\; P_{x1} > P_{x2}$$

$$\frac{m}{P_{x1}} < \frac{m}{P_{x2}} \;\; (\frac{m}{P_y} \text{ unchanged})$$

Demand curve is downward-sloping.

Demand curve is upward-sloping.

$\Rightarrow$ X is a Giffen good

P_x changes $\Rightarrow$ y^* changes .

$x^* = x\,(P_x, P_y, m)$

$y^* = y\,(P_x, P_y, m)$

In the previous analysis , we have $\frac{\partial x^*}{\partial m}$ and $\frac{\partial y^*}{\partial m}$ (Engel curves)

$\frac{\partial y^*}{\partial P_x} < 0$ X and Y are complements. (Y is a complement of X)

$\frac{\partial y^*}{\partial P_x} > 0$ X and Y are substitutes. (Y is a substitute of X)

$\frac{\partial y^*}{\partial P_x} = 0$ X and Y are not related.

EX： X and Y are perfect complements.

3 units of X is used with 2 units of Y.

$$u(x, y) = \min\{\frac{x}{3}, \frac{y}{2}\}$$

$$P_x\, x + P_y\, y = m$$

$$P_x\, x + P_y\, \frac{2}{3} x = m$$

$$3P_x\, x + 2P_y\, x = 3m$$

$$(3P_x + 2P_y)\, x = 3m$$

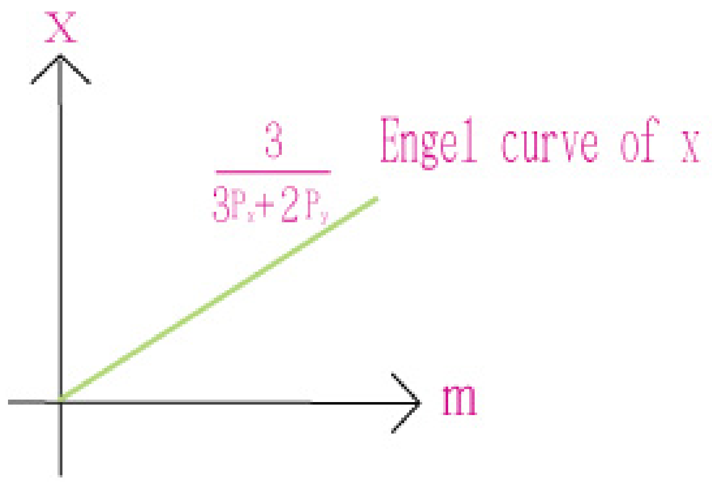
$$x^* = \frac{3m}{3P_x + 2P_y}$$

$$y^* = \frac{2m}{3P_x + 2P_y}$$

$$x = \frac{3}{3P_x + 2P_y} m$$

$$y = \frac{2}{3P_x + 2P_y} m$$

} Engle curves



F igure43 :Engel curve in this case

Demand curve of X

$$x = (3m) \frac{1}{(2P_y) + 3P_x}, \quad \frac{\partial x}{\partial P_x} < 0$$

fixed